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SOME PROBLEMS OF COMPUTER MATHEMATICS

- USSR -

by L. A. Lyusternik and S. L. Sobolov

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The past ten years have been marked by a rapid development of computer mathematics, but within the limits of a magazine article it is impossible to give a detailed summary of the present condition of this, now very real, branch of knowledge. We will attempt to reveal only some of what, from our standpoint, are the more characteristic tendencies in its development. The material which is introduced has illustrative significance and, certainly, the mention or non-mention of this or that work has not been caused by estimated criteria. It is natural also, that the selection of materials is connected with the scientific interests of the authors and the problems closest to us are elucidated in more detail.

The Expansion of Computer Mathematics. In practice, solutions of mathematical problems are usually satisfied by an approximate location of the object. This gives the right to approximately substitute the problem itself with another simpler problem for a numerical solution. With the development of mathematics both, the sought for objects and the problems themselves, have changed. First, in time, were the problems connected with an approximate location of numerical objects (for example, the determination of x, extraction of a root, solution of algebraic equations). The next stage was problems of approximation of functions, which were connected with a creation of apparatuses for presenting functions -- power series, continued fractions, orthogonal analysis, multiple and rational approximations and so forth. Practically simultaneously there arose problems which actually stand for approximate computations of functionals and operators -- numerical intergrations and differentiations, numerical solution of differential equations.

Until the 20th century the content of numerical analysis had basically come to an interpolation and variable apparatus and methods of numerical integration based on them. In the 20th century, a considerable broadening and complication of problems of numerical analysis is observed. To it were introduced the solution of marginal problems, integral equations, finding of proper values and elements and other characteristics of the spectrum and of conforming reflections and so

forth. This caused an appearance of new methods. The circle of problems is continually broadening because of the complication of the mathematical apparatus of modern physics, the appearance of new possibilities for computing and the rapid accumulation of experience of solutions of various problems which have actually been carried out.

The broadening of the field of application of mathematics and the circle of solved problems has led to a considerable increase in size and variety of computations carried out. Special mechanisms have been required which would permit these computations to be carried out in a short period.

The appearance of new means of computing techniques has been caused by the great inverse influence in the development of computer mathematics. A possibility has appeared for solving problems, the solution of which previously had been considered absolutely impossible. It is doubtful, for example, whether a solution would have appeared for a system of unknowns by an expression of three-digit ciphers. Equations of gas dynamics can be introduced as an example, These are systems of differential equations with a large number of variables and so forth. It is understandable that such highly precise calculations as the movement of space rockets and sputniks and the calculation of trajectories over a great distance, which must be made in very short periods, cannot be made without electronic computing machines.

It is natural that, in a short period of time, computer mathematics has changed its appearance both in the sense of methods and in the sense of breadth and range of solved problems.

In this situation of rapid growth, it would be very dangerous to approach the problems standing in this field and the preparation of specialists for solving these problems from the standpoint of the past.

During the 1920's and 1930's, if there sometimes arose a specialty of computer mathematics in universities it would come to a set of simple methods and to a study of standard solutions of problems leading to prescribed rules. There are some places where such a practice has not been completely overcome. A tendency still exists toward preparing specialists of computer mathematics along a narrow profile, without a knowledge of mathematics, in the broad sense of the word, at its modern level. Certainly, there is a need for such specialists who have a narrow profile, but experience has shown that it is better to prepare them over a short period of time from among those who have graduated from high school.

There are courses in operation for laboratory workers, computers and programmers in several organizations in Moscow. There are also such courses in Novosibirsk. The first schools have now been started in Moscow with a mathematician-computer inclination, in which, by the way, such specialists as computers and programmer-laboratory workers will be prepared. Obviously, a sufficiently broad network of such professional high schools and technical schools is needed. But universities must prepare mathematician-computers with a sufficiently broad and high mathematic culture, which is no less broad or high than the mathematicians

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There are courses in operation for laboratory workers, computers and programmers in several organizations in Moscow. There are also such courses in Novosibirsk. The first schools have now been started in Moscow with a mathematician-computer inclination, in which, by the way, such specialists as computers and programmer-laboratory workers will be prepared. Obviously, a sufficiently broad network of such professional high schools and technical schools is needed. But universities must prepare mathematician-computers with a sufficiently broad and high mathematic culture, which is no less broad or high than the mathematicians

of other specialities. Such mathematicians will represent the key staff of computer organizations. They must be used for preparing and programming the less standard computing problems and the more complicated problems of a logical nature. Unfortunately, people are found in our computer organizations, who have a mathematician's university diploma, but have not obtained a sufficiently broad and general mathematic preparation, and who carry out the very same functions as laboratory workers, computers and programmers with a high school education.

As an illustration of modern computer practice, we will introduce some statistical data from the Computer Center of the Moscow University. This Center possessed the machine "Strela" (the first of our series electronic computer machines), and at the present time it is also used for the electronic computer machine "Setun".

In the period 1957-1960, basically, such problems as non-linear differential equations of quotient derivatives, which are connected with problems of gas dynamics and hydro-dynamics, problems of expansion of electro magnetic fluctuations, problems of crystallography and chemistry which are reduced to a three-dimensional harmonic analysis, and so forth were solved on the "Strela". Among problems of linear algebra there were, for example, solutions of a system of 265 linear algebraic equations; among problems on plain differential equations there were solutions of n-point problems for non-linear equations of the nth order; in connection with the solution of some physics problems, a calculation of integrals in functional spaces was carried out etc.

In line with problems from high fields of mathematics, it would seem, completely elementary tasks were solved which concerned the selection of successful algorisms for the computation of elementary functions, on the machine "Strela", of methods for introducing arithmetic operations.

It would be a mistake to consider that the appearance of machines provides for the importance of analytical skill, the ability to obtain a solution in its final form etc. A combination of the skill of analytical transformation with the use of machines for conducting computations is necessary in modern physics research. A definite level of analytical culture is needed in order to interpret the cipher material produced by the machine, not to mention the successful selection of algorisms for the solution of problems.

Frequently algorisms are formulated in the process of operation, in the process of analyzing the produced material, in the process of diagnosing difficulties which arise, etc.

We know of examples of the successful solution on the machine "Strela" of problems of rational design of conveyences. Both on this machine and on the machine "M-2" experiments of games with incomplete information have been carried out. This includes such games as dominoes or cards, in which it is necessary to set up theories of probabilities concerning the resources of the "oponent", to check these theories on the basis of information on his procedures and to choose his procedures every time in the best rational form.

There is an ever broadening penetration of mathematical methods because of the application of modern machines in those fields of science and practice which very recently were considered to be far removed from mathematics or, at least, from non-elementary mathematics (economics, planning, technics). Because of this an organization of computer centers in the Gosplan USSR and the Gosplan RSFSR, of mathematician groups in sovnarkhozes, etc. has appeared.

Methods of functional analysis. The transformation of accumulated facts and processes and the approach to them from a single more gentle point of view is especially urgent in the conditions of the ever increasing size of computer tasks. Such a need arose several decades ago in various fields of mathematics and it led to the creation of those uniting conceptions which have come to be called functional analysis. Computer mathematics was one of the important sources for the formation of concepts of functional analysis. Its classical divisions were concerned with operators of differentiation and integration, displacement and differential ratio and the connections between these operators. Research of these connections was introduced as early as 100 years ago (Gauss, Koshi) toward the creation of a symbolic calculation which would consider the functions from the symbols of these operations. Thus the concept of operator and function from the operator first appeared.

In computer mathematics it was always necessary to be concerned with of certain functions with others and this meant that numerical characteristics of the proximity of functions were needed. With Chebyshev we already find various numerical characteristics of this approximation -- that which is now called the "distance" between functions. This distance was found in various ways depending on the nature of the problem. It was necessary to analogously consider the measures of proximity of the operators. Thus appeared the concept of metric distance, i.e. the great number of elements for which the concept of "distance" was introduced which satisfies the several properties of common distance. In analysis, and this includes approximation analysis, a particular role is played by functional spaces, of which functions, operators and functionals are elements.

The circumstance that methods of approximate solution of problems were applied in various fields of algebra and analysis has furthered the working out of uniting conceptions of functional analysis. For example, the method of simple integration for solution of algebraic and transcendental equations, Pikar's method for solving differential equations, Neiman's method for solving integral equations and others are joined by an integrational method for solving an equation in metric distances (i.e. in systems of elements for which the distance was defined).

Special criteria for the convergence of all such integrational methods are joined into a common criterian for the convergence of integrational processes in these distances (for example, in the "principle of compressed reflections").

And so we see that problems of approximate analysis played an essential role in the formation of conceptions of functional analysis. After their creation we observe a reverse influence of functional analysis on computer mathematics, which was revealed for the first time in the works of L. V. Kantorevich. We note that the diversity of problems in modern computer mathematics make an exposure of them with the uniting position of functional analysis advisable.

At the present time we can more clearly imagine a place for computer mathematics in a series of other mathematic disciplines.

For a clearer concept of possibilities and means of computer mathematics it is useful that a definition of a compact be introduced. Let a finite number of elements Mo be given in a metric space M so that each element of M has been extracted from at least one element of Mo for a distance less than E. Mo is called the finite E-system for M. If a finite system exists for any E which is greater than 0 then M is called a compact. A segment of a straight line is the simplest example of a compact. As it is well known, compacts play a large role in analysis.

The result of any computations is always expressed in the form of a finite quantity of finite numbers. The quantity of all conceivable computation results R2, among which the needed answer is indicated, is a finite number of ciphers. In contrast to this the problems being solved are usually contained in a finding of several elements from infinite functional spaces.

The number of conceivable answers for the problem being studied is most often an infinite quantity -- a functional space. A finite number R_n of conceivable calculation results must, with the exactness E necessary for us, representing the quantity R.

Accordingly, the range of problems, which are solved by numerical methods, is apparently limited by compact quantities. If R is a metric space, that means that the finite quantity R_n must represent the E-system for R. The work of Soviet mathematicians on the application of methods of functional analysis in the theory of approximate computations has already been elucidated in review articles of both the "TRUDAKH" of the 3rd Mathematic Congress (1956) and the conference on computer mathematics and technics (1959). We will, therefore, pause only on isolated examples. A series of basic methods for approximate solution of functional equations appeared as a result of a transfer, to common equations for various computer processes, of algebra and elementary analysis. This became possible as a result of the development of functional analysis, thanks particularly to the working out of an apparatus of analysis for linear operators and an apparatus of differential integral calculation for linear operators. Such a transfer of methods and the treatment common to them has not only broadened the field of their application, but has developed more fully their possibilities and the mechanism of their operation, so that new data concerning them, even in the case of algebra, have been obtained.

For example, the classical method of Newton's for solving algebraic equations is a means of consecutive approximations which make use of linearization. This method was transformed by L. V. Kantorovich and his students (G. P. Akilov, I. P. Mysovskikh and others) into a broad class of functional equations. The research by S. M. Lozinskiy on accuracy through the theory of implicit functions and his research on systems of differential equations are joined to this circle of problems.

Chaplygin's method, which is also extended to the general class of functional equations, is closely connected to Newton's method.

Problems of "stability" of algorisms play an essential role in modern computer practice. The question is whether the errors permitted in the process of computing will be "eliminated" or will "fluctuate" in the future. The errors of rounding off, from the one side, and the errors coming from substituting functions of operators and so forth by their approximations, from the other side, are inescapable. In the first case we have a stable algorithm. In the second case we have an unstable algorithm. It is natural that stability is useful as a rule. In particular, the criteria of stability is now the basic criteria in solving differential equations by a grid method and in a grid approximation. It was found expedient to conduct theoretical study of occurrences of stability of grid approximations by general methods of functional analysis as it was done by A. F. Filippov.

The numerical method of solving problems of analysis is somewhat of a distinct process, depending on some parameters (for example, spacing of grids in the differential method). In the tendency of the parameters toward natural limits (towards zero or infinity) this distinct process can be transformed into a continuous process. The ultimate continuous process (for example, in the case of Fredgolm's most "natural" method for numerical solution of integral equations) can contain peculiarities which are expressed in losses of a great number of signs.

As we have already said, computer mathematics is concerned with the approximation of compact quantities by finite quantities.

In the event that the considered objects are, in essence, elements of metrical space, the final system should be an E-system, i.e. it should approximate any element with an exactness to E. An important characteristic of a compact quantity is N(e) the dependence of the number of its elements N on the amount of expected errors. The possibility of such a prior appraisal of N(e) permits a consideration of the probability of obtaining a given answer to a problem being researched, and, consequently, the amount of necessary information.

Thus one may now approach a theoretical appraisal of that labor which must be spent on a solution of one problem or another with a given exactness.

Existing procedures and algorithms sometimes turn out to be too long, have too many operations and require great improvement. We are now, apparently, at the very beginning of a radical revision of views for a computer algorithm from this standpoint.

These ideas were expressed in the works of A. G. Vitushkin, N. S. Bakhvalov, A. N. Kolmogorov and other mathematicians. In the near future we will be studying old algorithms and creating new ones which have a number of operations closer to optimum. We have an example of such a new approach toward the future creation of optimum algorithms in the works of N. S. Bakhvalov on algorithms for the solution of a problem directly with the help of the grid method with the thickness of points increasing, with approximation, toward the limit.

Other examples of finding algorithms, which are close to optimum, are the new theories of number procedures for computing integrals worked out by N. M. Korobov and N. S. Bakhvalov.

It turns out that placing a point in a field, according to laws which take into account their arithmetic nature, can obtain almost the best algorithms for calculation of such integrals.

In connection with this it is very interesting to point out that the statistical method of "Monte Carlo", which is now being used with great popularity, offers a number of operations far from optimum in such cases when the field is a parallel piped and gives way to numerical operations.

At the present time, in numerical analysis, the transition to an approximate and numerical solution of problems of a more complex nature, which are advanced by the needs of application of an indirect apparatus of functional analysis, must stand in line. These problems belong not to separate elements of functional spaces, but to the analysis of such a space as a whole — to the study of one or another of its characteristics. The problem of numerical integration of so-called continual integrals through functional space can serve as an example. As we have already said, experiments on this type of computation have been conducted in the Computer Center of the Moscow University. Problems of approximation methods for restoring operators according to their spectr, which are important for theoretical physics, have begun to be exploited.

Computer Mathematics and Mathematical Logic. Its connection with problems of mathematical logic is another characteristic line of modern computer mathematics. Even in the thirties the theory of algorithms branched off from it. It considers, from a general theoretic position, the question of the realization of the solution of mathematical problems on automatically controlled machines (Tyuring machine). In connection with the creation of high speed mathematic machines, similar problems acquire great practical significance. With this, the role of man must consist basically in introducing the program into the machine, which compels the machine to carry out a succession of operations realizing the algorithm for the solution of a given problem. The program consists of a series of "commands" — definitely encoded "Instructions" — which compel the machine to select members from indicated sections which remember an arrangement ("memory"), carry out the given operation and send the answer to the indicated section ("memory"). A special chapter of applied mathematics — "programming" — has appeared, which is closely

joined with mathematic logic and the theory of algorithms. With this, practice has advanced new problems, which were not raised earlier in the theory of algorithms. The question did not simply concern the composition of a program for the solution of a given problem, but it also concerned the creation of a short program. We control the solution of a problem, requiring the fulfillment of millions of operations, with the help of a program from a hundred or a thousand commands, and if we were not able to do this then we would lose all of the advantages offered by high-speed machines.

The composition of programs, their control and so forth are a laborious and prolonged affair. Problems of the composition of programs which were worked out in our huge computing organizations, problems of deciphering programs (the restoration of mathematical problems according to their programs), problems of transformation of programs and so forth have appeared in connection with this. It is clear that these problems, which are concerned not with a concrete algorithm but with an algorithm in general, are solved on a higher level of abstraction.

A group of collectives worked in this direction under the direction of A. A. Lyapunov, M. P. Shura-Bura, N. A. Krynitskiy and others.

Universalizing those concepts and methods of thought which have appeared within mathematics is one of the important factors in the development of modern science and technics. Thus, the concept of algorithm -- a process of formalization -- has assumed universal significance. We have spoken of an algorithm for controlling a machine, an algorithm for translating from one language to another, an algorithm for operating a dispatcher, an algorithm for obtaining a dispatcher, an algorithm for obtaining information and so forth. The significance of such a generalization is the fact that machines, created for the realization of mathematic algorithms, have begun to be converted into machines for the realization of algorithms in general. Therefore it is very important to convert a given process into an algorithm, i.e., to formalize it. In connection with this, the formalization of processes for controlling production is extremely important. With this a machine carries out simultaneously both logically controlling functions and mathematic functions -- it solves problems located in the given conditions of optimum regime.

The concept of pattern has the same universal nature as the concept of algorithm.

The concept of pattern in science has two meanings which are, however, close in content. From the one side, for the application of mathematical methods towards the research of a given occurrence, it is necessary that this occurrence be schematized and simplified in a sense that it be separated from those sides of the given occurrence which do not play an essential role. For example, in the study of the movement of liquid in hydrodynamics, in many cases we can digress from the influence of viscidity, then the movement of an "ideal" liquid, without viscidity, will be the pattern. It is natural that a broadening of the

application of mathematics to new fields requires the creation of corresponding patterns of occurences which are studied in these fields of knowledge.

From the other side, the concept of pattern in mathematics is connected with the concept of isomorphism: one system of an axiom can have various realizations, various patterns which are isomorphous between themselves; objects and their correlations in one realization answer objects and their correlations in another realization. Every fact which is formed in one realization can agree as a correlative fact in another realization. This condition has been used for a long time in science and technics. For example, an isomorphism exists between the statics of a hard body and the kinematics of several adjoining systems.

The general concept of pattern has begun to play a large role in modern science and technics. Thus, identical equations describe various physical processes. Among these processes can be those which are easily realized and the results of which are easily measured. These processes can serve as patterns for all others, which are represented by the very same equations. As is known, such patterns are widely applied in technics. So-called analogy machines in computer technics are a peculiar case of similar patterns.

At the present time arithmetic logical patterns have begun to be applied, which are carried out on mathematic machines (and can be combined with a physical pattern). For example, in making a pattern of a random process on a machine, it can be combined with a physical pattern-generator of random impulses which are interpreted as ciphers of a random number. Now, for example, if we obtain, with this method, a pattern of a random amount with a uniform distribution; then by means of purely mathematical transformations it is easy to obtain patterns of random amounts with other laws of distribution (there also exist purely mathematical methods for approximately obtaining such amounts). A machine can be compelled to "trace" the course of a studied process, and, when the operation of a random amount comes into effect, to produce its random meaning. Since, in modern program, controlled machines, everything is carried out with great rapidity, then it is possible to carry out similar tests many times and to form a frequency of occurence of one result or another. With this it is necessary to take into account the fact that even if there is a possibility of carrying out an experiment on an actual object, then a highly shortened test is not always possible nor does it require a long time. Another highly valuable peculiarity of arithmetic-logical pattern making is the possibility of easily modifying the parameters of a process being patterned and of tracing the results of this modification. (We note that the pattern making by G. M. Adelson - Velskiy on the "M-2" machine of several processes of disintegration of mesons which are being studied in the Institute of Theoretical and Experimental Physics, has allowed errors of observation to be revealed.)

Mass processes can be modelled for which experimentation is naturely impossible or difficult. Thus, it is not difficult to represent, for example, an arithmetic-logical pattern of a plant or shop, which exists only in plan. The machine can trace the path of any detail. The "dispersion" of time spent on the operation of the machine is patterned by the introduction of corresponding random amounts and so forth. By means of a highly shortened repetition of such pattern making it is possible to form a frequency of one occurence or another, to establish "narro2" points and to trace the influence of any changes in the scheme of the work process. The perspectives for pattern making in various fields of life are exceedingly great. Logical-mathematic pattern making can combine with the physical also with the participation of actual objects.

In connection with the development of automatization and with the ever broadening transmission to machines of functions, which were previously performed by man, the principally important task of making patterns of the behavior of living beings has appeared. Pattern making of the process of training -- the accumulation of experience -- is particularly important in practice. For example, a program, which is introduced into a machine which controls several objects, can predict a statistical treatment of results of the machine's operation for the object and for the introduction into itself of changes on the basis of the results of this treatment. Here we enter into a very important new area of science -- cybernetics -- which is the general study of controlling processes.

Speaking of modern computer mathematics, its close connection with the theory of probability and statistics should be pointed out. We have already spoken about pattern making of random processes. As is known, a similar pattern making is indirectly applied toward the solution of mathematical problems. This is the so-called method of "Monte-Carlo". The amount, which is sought, is considered as a mathematic expectation of the result of any random process.

Speaking of the application of probability methods, of the "Monte-Carlo" type, toward inter-mathematic problems, it should be remembered that, in isolated cases, the very concept of problem solution must be improved.

In the classical understanding Xo -- the approximate value for the number x is such a value that the probability $P(X - Xo) > E$, whereby the true answer differs from X and is more than E, is equal to zero.

Sometimes, however, this is practically impossible and it is necessary to name a number $\geqslant 1 - \eta$ with an answer having an accuracy to E and with a probability P so that: $[P(X - Xo) > E] < n$ and it is often possible to find such an answer with the help of an "actual" number of operations, when at the same time, it is impossible to find an answer in a "classical" case (such a point of view is carried out in the works of V. S. Vladimirov).

Many important questions are not touched upon in the present articles, but is shown to indicate that the development of modern computer mathematics is closely connected with the most varied areas of mathematical sciences and, using their results in turn, is now rendering a highly inverse effect on the development of mathematics as a whole.

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